1. The distances travelled to work, D km, by the employees at a large company are normally distributed with $D \sim N(30, 8^2)$.

(a) Find the probability that a randomly selected employee has a journey to work of more than 20 km.

(3)

(b) Find the upper quartile, Q_3 , of D.

(3)

(c) Write down the lower quartile, Q_1 , of D.

(1)

An outlier is defined as any value of D such that $D < h$ or $D > k$ where

$$h = Q_1 - 1.5 \times (Q_3 - Q_1) \quad \text{and} \quad k = Q_3 + 1.5 \times (Q_3 - Q_1)$$

(d) Find the value of h and the value of k.

(2)

An employee is selected at random.

(e) Find the probability that the distance travelled to work by this employee is an outlier.

(3)
(Total 12 marks)

2. The heights of a population of women are normally distributed with mean μ cm and standard deviation σ cm. It is known that 30% of the women are taller than 172 cm and 5% are shorter than 154 cm.

(a) Sketch a diagram to show the distribution of heights represented by this information.

(3)

(b) Show that $\mu = 154 + 1.6449\sigma$.

(3)

(c) Obtain a second equation and hence find the value of μ and the value of σ.

(4)

A woman is chosen at random from the population.

(d) Find the probability that she is taller than 160 cm.

(3)
(Total 13 marks)

3. The lifetimes of bulbs used in a lamp are normally distributed.
A company X sells bulbs with a mean lifetime of 850 hours and a standard deviation of 50 hours.

(a) Find the probability of a bulb, from company X, having a lifetime of less than 830 hours.

(3)

(b) In a box of 500 bulbs, from company X, find the expected number having a lifetime of less than 830 hours.

(2)

A rival company Y sells bulbs with a mean lifetime of 860 hours and 20% of these bulbs have a lifetime of less than 818 hours.

(c) Find the standard deviation of the lifetimes of bulbs from company Y.

(4)

Both companies sell the bulbs for the same price.

(d) State which company you would recommend. Give reasons for your answer.

(2)
(Total 11 marks)

4. The random variable X has a normal distribution with mean 30 and standard deviation 5.

(a) Find $P(X < 39)$.

(2)

(b) Find the value of d such that $P(X < d) = 0.1151$

(4)

(c) Find the value of e such that $P(X > e) = 0.1151$

(2)

(d) Find $P(d < X < e)$.

(2)
(Total 10 marks)

5. A packing plant fills bags with cement. The weight X kg of a bag of cement can be modelled by a normal distribution with mean 50 kg and standard deviation 2 kg.

(a) Find $P(X > 53)$.

(3)

(b) Find the weight that is exceeded by 99% of the bags.

(5)

Three bags are selected at random.

(c) Find the probability that two weigh more than 53 kg and one weighs less than 53 kg.

(4)
(Total 12 marks)

6. The weights of bags of popcorn are normally distributed with mean of 200 g and 60% of all bags weighing between 190 g and 210 g.

(a) Write down the median weight of the bags of popcorn.

(1)

(b) Find the standard deviation of the weights of the bags of popcorn.

(5)

A shopkeeper finds that customers will complain if their bag of popcorn weighs less than 180 g.

(c) Find the probability that a customer will complain.

(3)
(Total 9 marks)

7. The random variable X has a normal distribution with mean 20 and standard deviation 4.

(a) Find $P(X > 25)$.

(3)

(b) Find the value of d such that $P(20 < X < d) = 0.4641$

(4)
(Total 7 marks)

8. The measure of intelligence, IQ, of a group of students is assumed to be Normally distributed with mean 100 and standard deviation 15.

(a) Find the probability that a student selected at random has an IQ less than 91.

(4)

The probability that a randomly selected student has an IQ of at least $100 + k$ is 0.2090.

(b) Find, to the nearest integer, the value of k.

(6)
(Total 10 marks)

9. From experience a high-jumper knows that he can clear a height of at least 1.78 m oncein 5 attempts. He also knows that he can clear a height of at least 1.65 m on 7 out of 10attempts.

Assuming that the heights the high-jumper can reach follow a Normal distribution,

(a) draw a sketch to illustrate the above information,

(3)

(b) find, to 3 decimal places, the mean and the standard deviation of the heights the high-jumper can reach,

(6)

(c) calculate the probability that he can jump at least 1.74 m.

(3)
(Total 12 marks)

10. The heights of a group of athletes are modelled by a normal distribution with mean 180 cm and standard deviation 5.2 cm. The weights of this group of athletes are modelled by a normal distribution with mean 85 kg and standard deviation 7.1 kg.

Find the probability that a randomly chosen athlete,

(a) is taller than 188 cm,

(3)

(b) weighs less than 97 kg.

(2)

(c) Assuming that for these athletes height and weight are independent, find the probability that a randomly chosen athlete is taller than 188 cm and weighs more than 97 kg.

(3)

(d) Comment on the assumption that height and weight are independent.

(1)
(Total 9 marks)

11. A scientist found that the time taken, M minutes, to carry out an experiment can be modelled by a normal random variable with mean 155 minutes and standard deviation 3.5 minutes.

Find

(a) P(M > 160).

(3)

(b) $P(150 \le M \le 157)$.

(4)

(c) the value of m, to 1 decimal place, such that $P(M \le m) = 0.30$.

(4)
(Total 11 marks)

12. The random variable X is normally distributed with mean 79 and variance 144.

Find

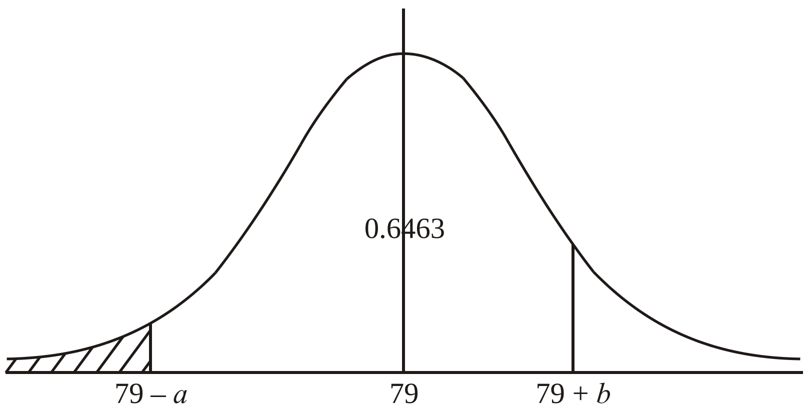
(a) $P(X < 70)$,

(3)

(b) $P(64 < X < 96)$.

(3)

It is known that $P(79 - a \le X \le 79 + b) = 0.6463$. This information is shown in the figure below.



Given that $P(X \ge 79 + b) = 2P(X \le 79 - a)$,

(c) show that the area of the shaded region is 0.1179.

(3)

(d) Find the value of b.

(4)
(Total 13 marks)

13. The random variable X ~ N(μ, σ^2).

It is known that

$P(X \leq 66) = 0.0359$ and $P(X \geq 81) = 0.1151$.

(a) In the space below, give a clearly labelled sketch to represent these probabilities on a Normal curve.

(1)

(b) (i) Show that the value of σ is 5.

(ii) Find the value of μ.

(8)

(c) Find $P(69 \leq X \leq 83)$.

(3)

(Total 12 marks)

14. Students in Mr Brawn's exercise class have to do press-ups and sit-ups. The number of press-ups x and the number of sit-ups y done by a random sample of 8 students are summarised below.

$$\Sigma x = 272, \quad \Sigma x^2 = 10\,164, \quad \Sigma xy = 11\,222,$$

$$\Sigma y = 320, \quad \Sigma y^2 = 13\,464.$$

(a) Evaluate S_{xx}, S_{yy} and S_{xy}.

(4)

(b) Calculate, to 3 decimal places, the product moment correlation coefficient between x and y.

(3)

(c) Give an interpretation of your coefficient.

(2)

(d) Calculate the mean and the standard deviation of the number of press-ups done by these students.

(4)

Mr Brawn assumes that the number of press-ups that can be done by any student can be modelled by a normal distribution with mean μ and standard deviation σ. Assuming that μ and σ. take the same values as those calculated in part (d),

(e) find the value of a such that $\mathrm{P}(\mu - a < X < \mu + a) = 0.95$.

(3)

(f) Comment on Mr Brawn's assumption of normality.

(2)
(Total 18 marks)

15. A health club lets members use, on each visit, its facilities for as long as they wish. The club's records suggest that the length of a visit can be modelled by a normal distribution with mean 90 minutes. Only 20% of members stay for more than 125 minutes.

(a) Find the standard deviation of the normal distribution.

(4)

(b) Find the probability that a visit lasts less than 25 minutes.

(3)

The club introduces a closing time of 10:00 pm. Tara arrives at the club at 8:00 pm.

(c) Explain whether or not this normal distribution is still a suitable model for the length of her visit.

(2)
(Total 9 marks)

16. The random variable X is normally distributed with mean μ and variance σ^2.

(*a*) Write down 3 properties of the distribution of X.

(3)

Given that $\mu = 27$ and $\sigma = 10$

(b) find $P(26 < X < 28)$.

(4)
(Total 7 marks)

17. Cooking sauces are sold in jars containing a stated weight of 500 g of sauce. The jars are filled by a machine. The actual weight of sauce in each jar is normally distributed with mean 505 g and standard deviation 10 g.

(a) (i) Find the probability of a jar containing less than the stated weight.

(ii) In a box of 30 jars, find the expected number of jars containing less than the stated weight.

(5)

The mean weight of sauce is changed so that 1% of the jars contain less than the stated weight. The standard deviation stays the same.

(b) Find the new mean weight of sauce.

(4)
(Total 9 marks)

18. The lifetimes of batteries used for a computer game have a mean of 12 hours and a standard deviation of 3 hours. Battery lifetimes may be assumed to be normally distributed.

Find the lifetime, t hours, of a battery such that 1 battery in 5 will have a lifetime longer than t.

(Total 6 marks)

19. A drinks machine dispenses coffee into cups. A sign on the machine indicates that each cup contains 50 ml of coffee. The machine actually dispenses a mean amount of 55 ml per cup and 10% of the cups contain less than the amount stated on the sign. Assuming that the amount of coffee dispensed into each cup is normally distributed find

(a) the standard deviation of the amount of coffee dispensed per cup in ml,

(4)

(b) the percentage of cups that contain more than 61 ml.

(3)

Following complaints, the owners of the machine make adjustments. Only 2.5% of cups now contain less than 50 ml. The standard deviation of the amount dispensed is reduced to 3 ml.

Assuming that the amount of coffee dispensed is still normally distributed,

(c) find the new mean amount of coffee per cup.

(4)
(Total 11 marks)

20. The weight of coffee in glass jars labelled 100 g is normally distributed with mean 101.80 g and standard deviation 0.72 g. The weight of an empty glass jar is normally distributed with mean 260.00 g and standard deviation 5.45 g. The weight of a glass jar is independent of the weight of the coffee it contains.

Find the probability that a randomly selected jar weighs less than 266 g and contains less than 100 g of coffee. Give your answer to 2 significant figures.

(Total 8 marks)

1. (a) $P(D > 20) = P\left(Z > \frac{20-30}{8}\right)$ M1

$= P(Z > -1.25)$ A1

$=$ **<u>0.8944</u>** **<u>awrt 0.894</u>** A1 3

<u>Note</u>

M1 for an attempt to standardise 20 or 40 using 30 and 8.
1st A1 for $z = \pm 1.25$
2nd A1 for awrt 0.894

(b) $P(D < Q_3) = 0.75$ so $\frac{Q_3 - 30}{8} = 0.67$ M1 B1

$Q_3 =$ **awrt <u>35.4</u>** A1 3

<u>Note</u>

M1 for $\frac{Q_3 - 30}{8}$ = to a z value

M0for 0.7734 on RHS.
B1 for (z value) between 0.67 ~ 0.675 seen.
M1B0A1 for use of $z = 0.68$ in correct expression with awrt 35.4

(c) $35.4 - 30 = 5.4$ so $Q_1 = 30 - 5.4$ = **awrt <u>24.6</u>** B1ft 1

<u>Note</u>

Follow through using their of quartile values.

(d) $Q_3 - Q_1 = 10.8$ so $1.5(Q_3 - Q_1) = 16.2$ so $Q_1 - 16.2 = h$
or $Q_3 + 16.2 = k$ M1

h=**<u>8.4 to 8.6</u>** and k= **<u>51.4 to 51.6</u>** both A1 2

<u>Note</u>

M1 for an attempt to calculate 1.5(IQR) and attempt to add or subtract using one of the formulae given in the question – follow through their quartiles

(e) $2P(D > 51.6) = 2P(Z > 2.7)$ M1

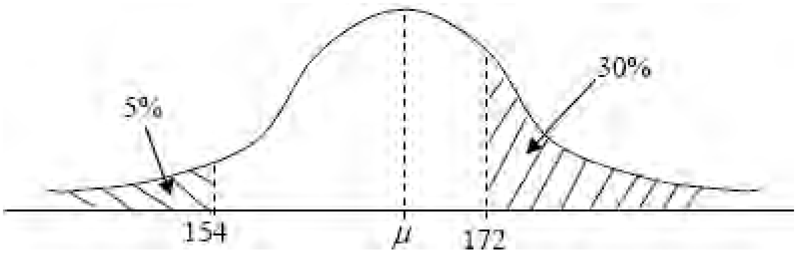
$= 2[1 - 0.9965]$ = awrt **<u>0.007</u>** M1 A1 3

<u>Note</u>

1st M1 for attempting $2P(D >$ their $k)$ or $(P(D >$their $k) + P(D <$ their $h))$
2nd M1 for standardising their h or k (may have missed the 2) so allow for standardising $P(D > 51.6)$ or $P(D < 8.4)$
Require boths Ms to award A mark.

[12]

2. (a)



bell shaped, must have inflexions	B1	
154,172 on axis	B1	
5% and 30%	B1	3

Note

2nd B1 for 154 and 172 marked but 154 must be $< \mu$ and $172 > \mu$. But μ need not be marked.

Allow for $\frac{154-\mu}{\sigma}$ and $\frac{172-\mu}{\sigma}$ marked on appropriate sides of the peak.

3rd B1 the 5% and 30% should be clearly indicated in the correct regions i.e. LH tail and RH tails.

(b) $P(X < 154) = 0.05$ M1

$\frac{154-\mu}{\sigma} = -1.6449$ or $\frac{\mu - 154}{\sigma} = 1.6449$ B1

$\mu = 154 + 1.6449\sigma$ * * given * * A1 cso 3

Note

M1 for $\pm \frac{(154-\mu)}{\sigma} = z$ value (z must be recognizable e.g. 1.64, 1.65, 1.96 but NOT 0.5199 etc)

B1 for $\pm$ 1.6449 seen in a line before the final answer.

A1cso for no incorrect statements (in μ, σ) equating a z value and a probability or incorrect signs e.g. $\frac{154-\mu}{\sigma} = 0.05$ or $\frac{154-\mu}{\sigma} = 1.6449$ or $P(Z < \frac{\mu-154}{\sigma}) = 1.6449$

(c) $172 - \mu = 0.5244\sigma$ or

$\frac{172-\mu}{\sigma} = 0.5244$ (allow z = 0.52 or better here but must be in an equation) B1

Solving gives $\sigma = 8.2976075$ (**awrt 8.30**) and $\mu = 167.64873$ (**awrt 168**) M1 A1 A1 4

<u>Note</u>

B1 for a correct 2nd equation (NB $172 - \mu = 0.525\sigma$ is B0, since z is incorrect)

M1 for solving their two linear equations leading to $\mu = ...$ or $\sigma = ...$

1st A1 for σ = awrt 8.30, 2nd A1 for μ = awrt 168
[NB the 168 can come from false working.

These A marks require use of correct equation from (b), and a z value for "0.5244" in (c)]

NB use of z = 0.52 will typically get σ =8.31 and μ = 167.67… and score B1M1A0A1

<u>No working</u> and both correct scores 4/4, only one correct scores 0/4

Provided the M1 is scored the A1s can be scored even with B0 (e.g. for z =0.525)

(d) P(Taller than 160cm) = $\mathrm{P}\left(Z > \dfrac{160-\mu}{\sigma}\right)$ M1

$= \mathrm{P}(Z < 0.9217994)$ B1

$= 0.8212$ **awrt 0.82** A1 3

<u>Note</u>

M1 for attempt to standardise with 160, their μ and their σ (> 0). Even allow with symbols μ and σ.

B1 for z = awrt $\pm$ 0.92

<u>No working</u> and a correct answer can score 3/3 provided σ and μ are correct to 2sf.

[13]

3. (a) Let the random variable X be the lifetime in hours of bulb

$\mathrm{P}(X < 830) = \mathrm{P}\left(Z < \dfrac{\pm(830-850)}{50}\right)$ Standardising with 850 and 50 M1

$= \mathrm{P}(Z < -0.4)$

$= 1 - \mathrm{P}(Z < 0.4)$ Using 1-(probability >0.5) M1

$= 1 - 0.6554$

$= 0.3446$ or 0.344578 by calculator awrt 0.345 A1 3

<u>Note</u>

If 1-z used e.g. 1 – 0.4=0.6 then award second M0

(b)	0.3446×500	Their (a) × 500	M1	
	$= 172.3$	Accept 172.3 or 172 or 173	A1	2
(c)	Standardise with 860 and σ and equate to z value $\frac{\pm(818-860)}{\sigma} = z$ value		M1	
	$\frac{818-860}{\sigma} = -0.84(16)$ or $\frac{860-818}{\sigma} = 0.84(16)$ or $\frac{902-860}{\sigma} = 0.84(16)$ or equiv.		A1	
	$\sigma = 49.9$	±0.8416(2)	B1	
		50 or awrt 49.9	A1	4

<u>Note</u>

M1 can be implied by correct line 2

A1 for completely correct statement or equivalent.

Award B1 if 0.8416(2) seen

Do not award final A1 if any errors in solution e.g. negative sign lost.

(d)	Company Y as the ***<u>mean</u>*** is greater for Y.	both	B1	
	They have (approximately) the same ***<u>standard deviation</u>*** or ***<u>sd</u>***		B1	2

<u>Note</u>

Must use statistical terms as underlined.

[11]

4.	(a)	$P(X < 39) = P\left(Z < \frac{39-30}{5}\right)$		M1	
		$= P(Z < 1.8) = \underline{0.9641}$	(allow awrt 0.964)	A1	2

<u>Note</u>

M1 for standardising with $\sigma, z = \pm\frac{39-30}{5}$ is OK

A1 for 0.9641 or awrt 0.964 but if they go on to calculate 1 – 0.9641 they get M1A0

(b) $P(X < d) = P\left(Z < \frac{d-30}{5}\right) = 0.1151$

$1 - 0.1151 = 0.8849$ M1

$\Rightarrow$ $z = -1.2$ (allow ± 1.2) B1

$\therefore \frac{d-30}{50} = -1.2$ $\underline{d = 24}$ M1A1 4

Note

1^{st} M1 for attempting 1– 0.1151. Must be seen in (b) in connection with finding d

B1 for z = ± 1.2. They must state z = ± 1.2 or imply it is a z value by its use.
This mark is only available in part (b).

2^{nd} M1 for $\left(\frac{d-30}{5}\right)$ = their negative z value (or equivalent)

(c) $P(X > e) = 0.1151$ so $e = \mu + (\mu - \text{their } d)$ or $\frac{e-30}{5} = 1.2$ or – their z M1

$e = 36$ A1 2

Note

M1 for a full method to find e. If they used $z = 1.2$ in (b) they can get M1 for $z = \pm 1.2$ here If they use symmetry about the mean μ + (μ – their d) then ft their d for M1 Must explicitly <u>see</u> the method used unless the answer is correct.

(d) $P(d < X < e) = 1 - 2 \times 0.1151$ M1
$= 0.7698$ AWRT $\underline{0.770}$ A1 2

Note

M1 for a complete method or use of a correct expression e.g. "their 0.8849" – 0.1151 <u>or</u> **If their *d* < their *e*** using their values with $P(X < e) - P(X < d)$ If their $d \geq$ their e then they can only score from an argument like 1 – 2x0.1151 A negative probability or probability > 1 for part (d) scores M0A0

[10]

5.	(a)	$z = \dfrac{53-50}{2}$	Attempt to standardise	M1	
		$P(X > 53) = 1 - P(Z < 1.5)$	1-probability required can be implied	B1	
		$= 1 - 0.9332$			
		$= 0.0668$		A1	3

M1 for using 53,50 and 2, either way around on numerator
B1 1– any probability for mark
A1 0.0668 cao

	(b)	$P(X \le x_0) = 0.01$		M1	
		$\dfrac{x_0 - 50}{2} = -2.3263$		M1B1	
		$x_0 = 45.3474$	awrt 45.3 or 45.4	M1A1	5

M1 can be implied or seen in a diagram
or equivalent with correct use of 0.01 or 0.99
M1 for attempt to standardise with 50 and 2 numerator either way around
B1 for ± 2.3263
M1 Equate expression with 50 and 2 to a z value to form an equation with consistent signs and attempt to solve
A1 awrt 45.3 or 45.4

	(c)	P(2 weigh more than 53kg and 1 less) = $3 \times 0.0668^2\ (1 - 0.0668)$		B1M1A1ft	
		$= 0.012492487..$	awrt 0.012	A1	4

B1 for 3,
M1 $p^2(1-p)$ for any value of p
A1ft for p is their answer to part (a) without 3
A1 awrt 0.012 or 0.0125

[12]

6.	(a)	200 or 200g		B1	1

"mean = 200g" is B0 but "median = 200" or just "200" alone is B1

	(b)	$P(190 < X < 210) = 0.6$ or $P(X < 210) = 0.8$			
		or $P(X > 210) = 0.2$ or diagram (o.e.)		M1	
			Correct use of 0.8 or 0.2	A1	
		$Z = (\pm)\dfrac{210-200}{\sigma}$		M1	
		$\dfrac{10}{\sigma} = 0.8416$	0.8416	B1	
		$\sigma = 11.882129....$	AWRT 11.9	A1	5

1st M1	for a correct probability statement (as given or eg P($200 < X < 210$) = 0.3 o.e.) or shaded diagram – must have values on z-axis and probability areas shown		
1st A1	for correct use of 0.8 or $p = 0.2$. Need a correct probability statement. May be implied by a suitable value for z seen (e.g. $z = 0.84$)		
2nd M1	for attempting to standardise. Values for x and μ used in formula. Don't need $z =$ for this M1 nor a z-value, just mark standardization.		
B1	for $z = 0.8416$ (or better) [$z = 0.84$ usually just loses this mark in (a)]		
2nd A1	for AWRT 11.9		

(c) $P(X < 180) = P\left(Z < \frac{180 - 200}{\sigma}\right)$ M1

$= P(Z < -1.6832)$

$= 1 - 0.9535$ M1

$= 0.0465$ or AWRT 0.046 A1 3

1st M1	for attempting to Standardise with 200 and their sd(>0) e.g. $(\pm)\frac{180-200}{\text{their } \sigma}$
2nd M1	**NB on epen this is an A mark ignore and treat it as 2nd M1** for 1 – a probability from tables provided compatible with their probability statement.
A1	for 0.0465 or AWRT 0.046 (Dependent on both Ms in part (c))

Standardization in (b) and (c). They must use σ not σ^2 or $\sqrt{\sigma}$.

[9]

7. (a) $P(X > 25) = P\left(Z > \frac{25-20}{4}\right)$ M1

$= P(Z > 1.25)$ M1

$= 1 - 0.8944$

$= 0.1056$ A1 3

Standardised with 20 and 4 for M1, allow numerator 20 – 25
1- probability for second M1
Anything that rounds to 0.106 for A1.
Correct answer with no working award 3/3

(b)	P(X < 20) = 0.5 **so** P(X < d) = 0.5 + 0.4641 = 0.9641		B1	
	P(Z < z) = 0.9641, z = 1.80		B1	
	$\frac{d-20}{4} = 1.80$		M1	
	d = 27.2		A1	4

0.9641 seen or implied by 1.80 for B1
1.80 seen for B1
Standardised with 20 and 4 and equate to z value for M1
Z = 0.8315 is M0
Anything that rounds to 27.2 for final A1.
Correct answer with no working 4/4

[7]

8.	(a)	P(X < 91) = P(Z < $\frac{91-100}{15}$)	Attempt standardisation	M1	
		= P(Z < – 0.6)		A1	
		= 1 – 0.7257		M1	
		= 0.2743	awrt **<u>0.274</u>**	A1	4

1st M1 for attempting standardisation. $\pm\frac{(91-\mu)}{\sigma \text{ or } \sigma^2}$.
Can use of 109 instead of 91.Use of 90.5 etc is M0

1st A1 for – 0.6 (or + 0.6 if using 109)

2nd M1 for 1 – probability from tables. Probability should be > 0.5)

(b)	1 – 0.2090 = 0.7910	0.791	B1	
	P(X > 100 + k) = 0.2090 <u>or</u> P(X < 100 + k) = 0.7910 (May be implied)		M1	
		Use of tables to get z = 0.81	B1	
	$\frac{100+k-100}{15}$, = 0.81 (ft their z = 0.81, but must be z not prob.)		M1, A1ft	
	<u>k = 12</u>		A1cao	6

1st B1 for 0.791 seen or implied.

1st M1 for a correct probability statement, but must use X or Z correctly. Shown on diagram is OK

2nd B1 for awrt 0.81 seen (or implied by correct answer – see below) (Calculator gives 0.80989…)

2nd M1 for attempting to standardise e.g. $\frac{100+k-100}{15}$ or $\frac{k}{15}$

$\frac{X-100}{15}$ scores 2nd M0 until the 100+ k is substituted to give k, but may imply 1st M1 *if* k = 112.15 seen

1st A1ft for correct equation for k (as written or better). Can be implied by k = 12.15 (or better)

2nd A1 for k = 12 only.

<u>Answers only</u>

k = 112 or 112.15 or better scores 3/6 (on EPEN give first 3 marks)

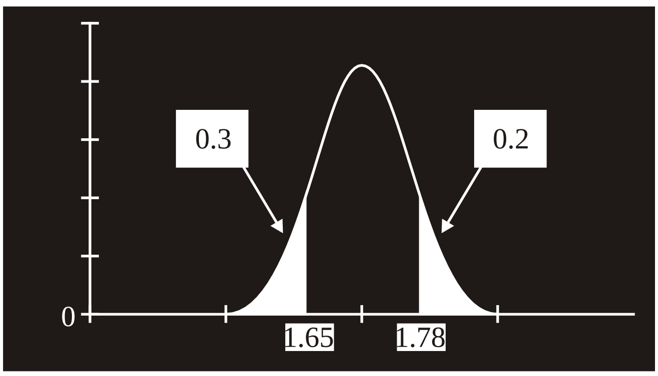
k = 12.15 or better (calculator gives 12.148438…) scores 5/6 (i.e loses last A1 only)

k = 12 (no incorrect working seen) scores 6/6

Using 0.7910 instead of 0.81 gives 11.865 which might be rounded to 12. This should score no more than B1M1B0M1A0A0.

[10]

9. (a)



2 separate sketches OK.

Bell shape	B1	
1.78 & 0.2	B1	
1.65 & 0.3	B1	3

Accept clear alternatives to 0.3; 0.7 / 0.5 / 0.2

(b) $\frac{1.78-\mu}{\sigma}=0.8416 \Rightarrow 1.78-\mu=0.8416\sigma$ M1

either for method

0.8416 B1

$\frac{1.65-\mu}{\sigma}=-0.5244 \Rightarrow 1.65-\mu=-0.5244\sigma$ B1

(–)0.5244

N.B. awrt 0.84, 0.52 B1B0

awrt 1.7, 0.095 cao M1 A1 A1 6

(c) P(height ≥ 1.74) = 1 – P (height < 1.74) M1

'one minus'

$=1-\mathrm{P}\left(\mathrm{Z}<\frac{1.74-1.70}{0.095}\right)$ M1

standardise with their mu and sigma

$=1-P(Z<0.42)=0.3372$ A1 3

awrt 0.337

[12]

10. (a) Let H be rv height of athletes, so $H \sim \mathrm{N}(180, 5.2^2)$

$\mathrm{P}(H > 188) = \mathrm{P}(Z > \frac{188-180}{5.2}) = \mathrm{P}(Z > 1.54) = 0.0618$

± stand √, sq, awrt 0.062 M1 A1 A1 3

(b) Let W be rv weight of athletes, so $W \sim \mathrm{N}(85, 7.1^2)$ M1 A1 2

$\mathrm{P}(W < 97) = \mathrm{P}(Z < 1.69) = 0.9545$ standardise, awrt 0.9545 M1 A1ft

(c) $\mathrm{P}(H > 188 \;\&\; W < 97) = 0.0618(1–0.9545)$ allow (a)×(b) for M A1 3

$= 0.00281$ awrt 0.0028

(d) Evidence suggests height and weight are positively correlated / linked

Assumption of independence is not sensible B1 1

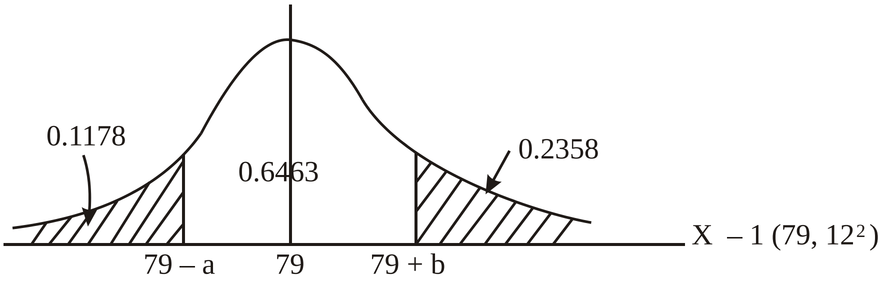
[9]

11.	(a)	$M \sim N(155, 3.5^2)$		
		$P(M > 160) = P\left(z > \dfrac{160-155}{3.5}\right)$	M1	
		standardising $\pm(160-155)$, σ, σ^2, $\sqrt{\sigma}$		
		$= P(z > 1.43)$	A1	
		$= 0.0764$	a1	3
	(b)	$P(150 \le M \le 157) = P(-1.43 \le z \le 0.57)$	B1 B1	
		awrt –1.43, 0.57		
		$= 0.7157 - (1 - 0.9236)$	M1	
		p > 0.5		
		$= 0.6393$	A1	4
		0.6393 – 0.6400 4dp		
		Special case: answer only B0 B0 M1 A1		
	(c)	$P(M \le m) = 0.3 \Rightarrow \dfrac{m-155}{3.5} = -0.5244$	B1 M1 A1	
		–0.5244		
		att stand = z value		
		for A1 may use awrt to –0.52.		
		$m = 153.2$	cao A1	4

[11]

12.	(a)	$P(X < 70) = P(Z < \dfrac{70-79}{12})$	M1	
		standardise 79, 12 or 79, 144		
		$= P(Z < -0.75) = 0.2266$	A1A1	3
		+ or -0.75, 0.2266		
	(b)	$P(64 < X < 96) = P(\dfrac{64-79}{12} < Z < \dfrac{96-79}{12})$	M1	
		standardise both, 79& 12 only		
		+ or –1.25 & 1.42, 0.8166	A1,A1	3
		Accept 0.8160-0.8170		

(c)



Shaded area = $\frac{1}{3}(1 - 0.6463)$ M1A1

=0.1179 cso A1 3

(d) P($X \le 79 + b$) = 0.7642 B1 implied

0.7642

$\Rightarrow \frac{b}{12} = 0.72$ M1A1

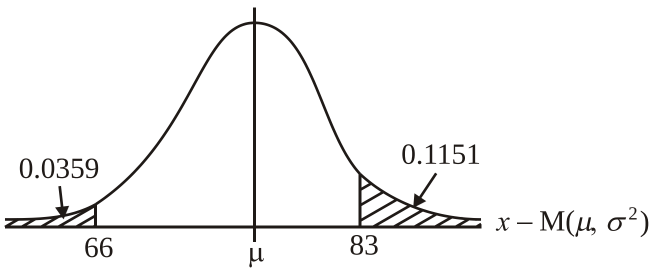
standardise LHS = z-value, all correct

b = 8.64 A1 4

3sf

[13]

13. (a)



Bell shaped curve & 4 values B1 1

(b) (i) $P\left(Z \le \frac{66-\mu}{\sigma}\right) = 0.0359 \Rightarrow 66 - \mu = -1.80\sigma$ –1.80 B1 seen

Clear attempt including standardization either way, or equivalentM1,A1

$81 - \mu = 1.20\sigma$ 1.20, or equivalent B1A1

Subtracting $15 = 1.20\sigma + 1.80\sigma \Rightarrow \sigma = 5$ **given answer*

Clear attempt to solve, cso M1A1

$\mu = 66 + 1.8 \times 5 = 75$ B1 8

(c) $P(69 \le X \le 83) = P\left(\frac{69-75}{5} \le Z \le \frac{83-75}{5}\right)$ standardize both either way M1

$= P(-1.20 \le Z \le 1.60)$ $-1.20, 1.60$ A1 seen

$= 0.8301$ 4 dp A1 3

[12]

14. (a) $S_{xx} = 10164 - \frac{272^2}{8} = 916$ M1,A1

Any one method, cao

$S_{yy} = 13464 - \frac{320^2}{8} = 664$ A1

cao

$S_{xy} = 11222 - \frac{272 \times 320}{8} = 342$ A1 4

cao

(**Or** 114.5,83 & 42.75)

(b) $r = \frac{342}{\sqrt{916 \times 664}} = 0.43852$ M1A1ftA1 3

formula, all correct ($\sqrt{608224}$), *0.439*

(c) Slight / weak evidence, B1

students perform similarly in pressups and situps B1 2

context for +ve

(d) $\bar{x} = \frac{272}{8} = 34$ M1A1

$s = \sqrt{\frac{10164}{8} - 34^2} = \sqrt{114.5} = 10.700$ M1A1 4

method includes $\sqrt{}$, *awrt 10.7*

OR *divisor (n – 1) awrt 11.4*

(e) $a = 1.96 \times 10.700\ldots = 20.9729$ (or 22.4 divisor (n – 1)) 1.96B1

$1.96 \times s$, 21.0 or 22.4 M1A1 3

(f)	Pressups discrete, Normal continuous	B1		
	Not a very good assumption	B1 dep	2	
				[18]

15. Let L represent length of visit $\therefore L \sim \text{N}(90, \sigma^2)$

(a)	$\text{P}(L < 125) = 0.80$ or $\text{P}(L > 125) = 0.20$		
	$\therefore \text{P}\left(Z < \frac{125-90}{\sigma}\right) = 0.8 \;\; \therefore \text{P}\left(Z > \frac{125-90}{\sigma}\right) = 0.20$	M1	
	Standardising $\pm(125-90)$, $\sigma/\sigma^2/\sqrt{\sigma}$		
	$\therefore \frac{125-90}{\sigma} = 0.8416$	B1	
	$\frac{\pm(125-90)}{\sigma} = z$ value	M1	
	$\therefore \sigma = \frac{35}{0.8416} = \underline{41.587\ldots..}$	A1	4
	AWRT 41.6		
(b)	$\text{P}(L < 25) = \text{P}\left(Z < \frac{25-90}{41.587....}\right)$	M1	
	Standardising 25, 90, their σ +ve		
	$= \text{P}(Z < -1.56)$		
	$= 1 - \text{P}(Z < 1.56)$	M1	
	For use of symmetry or $\Phi(-z) = 1 - \Phi(z)$; $p < 0.5$		
	$= \underline{0.0594}$	A1	3
(c)	Normal is not suitable		
	$90 + 2\sigma = 173.\dot{3} \Rightarrow$ 7.07 pm for latest arrival	B1	
	Comment based on $2\sigma/3\sigma$ rule		
	$90 + 3\sigma = 215 \Rightarrow$ 6.25 pm for latest arrival	B1	2
			[9]

16.

(a)	Symmetrical (about the mean μ) Mode = mean = median Horizontal axis asymptotic to curve	B1;B1;B1	3
	Distribution is ‘bell shaped’ – accept sketch 95% of data lies within 2 sd’s of the mean		
	Any 3 sensible properties		

(b) X ~ N $(27,10^2)$

$\therefore$ P $(26 < x < 28)$ = P$\left(\frac{26-27}{10} < Z < \frac{28-27}{10}\right)$

Standardising with $\mu = 27$, M1

σ *= 10 or $\sqrt{10}$* A1

One correct (seen)

= P$(-0.1 < Z < 0.1)$ -0.1 or 0.1	A1	
= Φ (0.1) – {1 – Φ(0.1)}		
or 2 × {Φ(0.1) – 0.5}		
= <u>0.0796</u> <u>0.0796 or 0.0797</u>	A1	4

Data is continuous	B0
Area under curve = 1	B0
Limits are $-\infty$ & ∞	B0
IQR contains 50% of data	B0
68% between $\mu \pm \sigma$	B1
Most of data within 3 s.d of mean	B1
No +ve or –ve skew	B1
Never touches axes at either side *(ie asymptotic)*	B1

17. (a) (i) Let X represent amount of sauce in a jar.

$\therefore X \sim$ N $(505, 10^2)$

$\therefore$ P$(X < 500)$ = P$\left(Z < \frac{|500-505|}{10}\right)$ M1

Standardising with 505, 10

= P$(Z < -0.5)$ A1

–0.5

= 1 – 0.6915

<u>= 0.3085</u> A1

(ii) Expected number = 30 × 0.3085 M1

30 × (i)

= <u>9.255</u> *or* 9.26 *or* 9.3 A1 5

(b) $P(X < 500) = 0.01$ A1

Or clearly labelled diagram
500 & 0.01 marked.

$\therefore \frac{500 - \mu}{10} = -2.3263$ M1

Standardising 500, 10, z-value

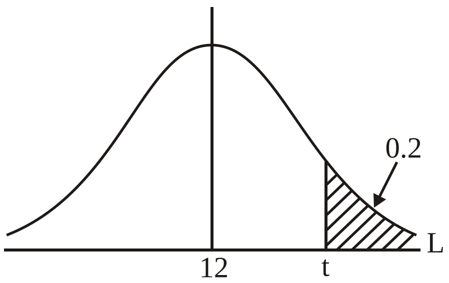
-2.3263 B1

$\underline{\therefore \mu = 523.263}$ A1 4

3 s.f. 523

[9]

18.



Let L represent lifetimes $\therefore L \sim N(12, 3^2)$

$P(L > t) = 0.2$ or $P(Z > \frac{t-12}{3}) = 0.2$ or diagram M1

$\therefore \frac{t-12}{3} = 0.8416$

$\therefore \underline{t = 14.5248}$

*0.8146*B1

Allow σ, σ^2 $\sqrt{\sigma}$ $\frac{t-\mu}{\sigma} = \delta$ M1

all correct A1
solving M1
14.5 A1 6

[6]

Alternative

P($L > t$) = 0.2 M1

$\therefore$ P($L \le t$) = 0.8

$\therefore$ $\frac{t-12}{3}$ = 0.84(18)

$\therefore$ $\underline{t = 14.52(14.5254)}$

0.84(18) B1

$\frac{t-12}{3}$ *= 0.84(18)* A1

solving M1

14.5 A1

19. Let X represent amount dispersed into cups

$\therefore$ $X \sim \text{N}(55, \sigma)$

(a) P(X < 50) = 0.10 $\Rightarrow$ $\frac{50-55}{\sigma}$ = –1.2816 M1 B1

σ = 3.90137 M1 A1 4

(b) P(X > 61) = P($Z > \frac{61-55}{3.90137...}$) M1

= P(Z > 1.54) A1

= 1 – 0.90382 = 0.0618; 6.18% A1 3

(c) Let Y represent new amount dispensed.

$\therefore$ $Y \sim \text{N}(\mu, 3)$

P(Y < 50) = 0.025 $\Rightarrow$ $\frac{50-\mu}{3}$ = –1.96 M1 B1

μ = 55.88 M1 A1 4

[11]

20. Let J represent the weight of a Jar $\therefore$ $J \sim \text{N}(260.00, 5.45^2)$

$\therefore$ P(J < 266) = $\text{P}\left(Z < \frac{266-260}{5.45}\right)$ M1 A1

= P(Z < 1.10)

= 0.8643 A1

(NB: calculator gives 0.86453: accept 0.864 – 0.865)

Let C represent weight of coffee in a Jar $\therefore\ C \sim \text{N}(101.8, 0.72^2)$

$\therefore\ \text{P}(C < 100) = \text{P}\left(Z < \dfrac{100 - 101.8}{0.72}\right)$ M1 A1

$= \text{P}(Z < -2.50)$ A1

$= 0.0062$

$\therefore\ \text{P}(J < 266 \ \& \ C < 100) = 0.8643 \times 0.0062$ M1

$= 0.0054$ A1 8

[8]

1. This question proved to be quite challenging for a high proportion of candidates. A significant number either made no attempt at the question or offered very little in the way of creditable solutions, with many unable to progress beyond part (a). Time issues may have been a contributing factor in some cases.

The majority of candidates however, were able to earn some credit at least in part (a), for their standardisation, although whilst this was often completely correct, a fairly common mistake was to give 1 – 0.8944 = 0.1056 as their final answer.

Many students did not recognise that they needed to actually use the normal distribution in part (b) and part (c), giving rise to extremely poor attempts by numerous candidates. Of these, many merely gave 45 and 15 as their quartiles, whilst others calculated $\frac{3}{4}$ of some value as their upper quartile (for example $\frac{3}{4} \times 60$) and $\frac{1}{4}$ of the same value as their lower quartile. Alternatively, of those who understood that they were required to use the normal distribution, most attempts were successful, though there were some instances of their setting their standardisation equal to a probability, usually 0.75 or $P(Z < 0.75)$, and not a z-value. Unfortunately 0.68 was used fairly frequently as the z value. The majority of candidates were however able to follow through their value of the upper quartile to find their lower quartile using symmetry, though some performed a second calculation involving standardisation. Some candidates miscalculated their lower quartile as $\frac{1}{3}$ of their upper quartile.

Despite previous errors most candidates tended to be successful in substituting their values correctly into at least one of the given formulae. However, a few seemed unaware of the order of the operations.

The final part of the question also proved difficult for many candidates with some running into trouble as a consequence of previous errors in part (b), part (c) and part (d) and others providing no attempt at all. Indeed, for numerous candidates, incorrect values for h and k led to probabilities of 0 being calculated from results such as $P(Z > 7)$ and thus many creditable attempts lost marks through earlier inaccuracies.

2. Part (a) was usually answered well but the remaining parts of the question proved challenging for many. There was much muddled work in part (b) and although some scored M1B1 for attempts such as $\frac{154-\mu}{\sigma}$ = 1.6449 very few scored the A1cso for a completely correct derivation without any incorrect statements being seen. Those who fumbled their way to the printed answer in (b) usually came unstuck in part (c). A common error was to write $\frac{172-\mu}{\sigma}$ = 0.5244 and then replace he 0.5244 with 1 – 0.5244. Those with a correct pair of equations were usually able to solve them correctly to find μ and σ. Many attempted to standardise in part (d) but even those with correct answers in (c) often failed to score full marks either due to premature rounding or because they thought their final answer was 1 – 0.8212. Curiously even a correct diagram failed to prevent some of them from making this final error.

3. More able candidates made a good start to this question. Part (a) was well done and part (b) usually gained the method mark. Part (c) proved to be much more of a challenge despite it being very similar to questions set in previous papers on this topic. Many candidates gained 4 marks, and there were a number who could see what was required but could not quite answer fully, submitting solutions which had most of the components, but not in the right sequence. Many adjusted the sign, either losing it during the calculation, or right at the end when –50 did not appear to be correct. Many candidates did not use 0.8416, settling for 0.84. A few used a probability rather than a z value but this was less than in previous years. Some candidates drew diagrams to help their thought processes. In part (d) candidates lost marks as they were not confident in interpreting what their figures meant. Many candidates did not use correct statistical language and thus lost the marks, others commented on the standard deviation for Y being lower than that for X without considering the magnitude of the difference.

4. Most candidates tried this question and the standardisation in part (a) was usually correct but a small minority used 25 as the standard deviation. The majority found $\mathrm{P}(Z < 1.8)$ correctly but some gave the answer as 1 – 0.9641 and lost the second mark.

A clear diagram should have helped candidates with the next two parts for many gave answers to d and e where $d > e$. In part (b) many started correctly by calculating 1 – 0.1151 and using the tables to find $z = \pm 1.2$. However only the more alert chose the minus sign and they usually went on to score full marks in both parts (b) and (c). There were good arguments using the symmetry of the normal distribution in both parts (c) and (d). Some candidates who made little progress with (b) or (c) were able to draw a simple diagram in (d) and obtain the correct answer from 1 – 2 × 0.1151.

5. Part (a) was answered with the highest degree of success with all but the weakest students not gaining 3 marks. Many candidates incorrectly interpreted the sign of the inequality in part (b) and went on to calculate a score above the mean. Many of those who arrived at an answer got circa 54 kg and failed to consider the reasonableness of their answer i.e. if the mean is 50kg, then 99% of the packets cannot weigh more than 54kg. Candidates often did not use the percentage points table quoting 2.33 instead of 2.3263 and this was reflected in the very small proportion of B marks awarded in part (b). Few candidates equated their standardised equation to a z-score with a consistent sign. Part (c) really sorted out those who really understood what was going on and who were hazily following the rules. Many failed to recognize that they only required the answer from part (a), a marked number specifically calculating the individual probabilities anew. The number who recognized that there was a factor of three involved was small although a significant number scored 2 marks by using $p^2(1 - p)$. Quite marked was the significant number who multiplied 0.0688 by two rather than squaring it. There was also a fair number who added the probabilities rather than multiplying.

6. Most candidates knew that mean = median for a normal distribution and wrote down the correct value, others obtained this by calculating (190 + 210)/2. In part (b) many were able to illustrate a correct probability statement on a diagram and most knew how to standardize but the key was to identify the statement $\mathrm{P}(X < 210) = 0.8$ (or equivalent) and then use the tables to find the z value of 0.8416 and this step defeated the majority. Some used the “large” table and obtained the less accurate $z = 0.84$ but this still enabled them to score all the marks except the B1 for quoting 0.8416 from tables. In part (c) most were able to score some method marks for standardizing using their value of σ (provided this was positive!) and then attempting 1 - the probability from the tables. As usual the candidates’ use of the notation connected with a normal distribution was poor: probabilities and z values were frequently muddled.

7. Part (a) was often done well but a substantial number omitted to subtract from 1; however most attempts standardised correctly with 4. The majority of candidates found part (b) required too much reasoning and either failed to add the 0.5 or got completely muddled in the use of the normal tables. A significant number did not understand the difference between a probability and a z value. A minority of candidates did not attempt this question.

8. Apart from the small minority who used σ^2 or $\sqrt{\sigma}$ in their standardisation, this part of the question was answered well. A common mistake in part (b) was to think that P($X < k + 100$) = 0.2090. The use of notation was often poor (with z values and probabilities often being equated) but many were able to find 0.7910 (from 1 – 0.2090) and often they also found $z = 0.81$ although a few rounded the 0.2090 to 0.20 and used $z = 0.8416$ from the table of percentage points. A number failed to standardize correctly and left the answer as $k = 112.5$ and others forgot that k was required to the nearest integer and left their answer as $k = 12.15$. Overall though this question was answered quite well.

9. It was unusual if a candidate scored 3 marks for the sketch. The mark for a bell-shaped curve was awarded to most candidates, but a particularly common problem was putting the value 1.65 on the wrong side of the sketch. Putting enough correct probabilities in the spaces was not

10. A straight forward Normal question answered well by some candidates and not so well by others. Part (d) proved a good discriminator between those who understood the meaning of independence and those who didn't, though some were sidetracked into trying to justify the assumption rather than questioning it with some strange results.

11. (a) This part of the question was generally well answered, with only a few candidates attempting to standardize with 3.5^2 or $\sqrt{3.5}$. Some candidates were unable to calculate the required probability once 0.9236 had been obtained. Occasionally a truncated value of z = 1.42 resulted in the final accuracy mark being lost.

(b) Many correct solutions were seen here.

(c) The majority of candidates seem to be unaware of the use of the percentage points table, and it was relatively rare to see z = - 0.5244. The common errors were to use the tables incorrectly and use a value of 0.6179 or simply to use 0.3 or 0.7.

12. The best candidates picked up full marks for this question. Generally part (a) and part (b) were answered well. There were many longwinded solutions to part (c) and quite a few confused responses to part (d) with confusion between z-scores and probabilities. Most candidates can standardise and find probabilities correctly, although some still use variance instead of standard deviation. Many candidates missed the simplicity of part (c) trying to over complicate it, and most of these never attempted part (d), perhaps not realising that they did not require part (c) for part (d).

13. A poorly answered question, with a significant minority having little if any familiarity with normal distributions. The sketch was nearly always the correct shape, although the four values of 66, 81, 0.0359 and 0.1151 were not always indicated on it. Some thought that a probability of 0.0359 corresponded to over one half of the area beneath the curve. Confusion between probabilities and z-values is still extremely common, The first z-value of -1.80 was often not found; 1.80 was common, with some candidates faking their standardisation equation to eventually obtain the given value of s = 5, whilst others were unable to obtain this value at all. Candidates were more successful in obtaining the second z-value of 1.20. Attempts at solving the simultaneous equations were usually satisfactory. It was surprising to see how many candidates used their incorrect value of s to calculate the value of μ. In part (c), the standardisation was usually done well by those candidates who reached this stage, with the correct answer frequently being obtained.

14. Parts (a) and (b) were extremely well answered by candidates; the value of 664 for S_{yy} was occasionally miscopied as 646 from part (a) to part (b). Candidates found it surprisingly difficult to obtain both marks in part (c), with a contextual relationship frequently being omitted. In part (d) the calculation of the mean was straightforward for nearly all candidates. Those candidates who were able to provide a correct formula also accurately found the standard deviation; however, too many candidates at this level were quoting an incorrect formula. Part (e) proved a good discriminator, with relatively few concise solutions; some candidates managed to obtain the correct value of a after a page or so of working. Only a handful of candidates were able to see that the number of press-ups is a discrete variable, whereas normal distributions are continuous.

15. The use of the Normal distribution appeared to be understood this year, but as before many of the candidates did not use the tables accurately in part (a). In spite of previous advice many candidates used a z-value of 0.84 instead of 0.8416 with consequent loss of marks. Part (b) was well answered but very few candidates could give a reasonable answer to the final part.

16. There were very few good descriptions of the properties of the Normal distribution. Many candidates made comments that were general to any continuous probability distribution rather than specific to the Normal distribution. A generous mark scheme allowed many candidates to score most of the marks in part (b) but many could not handle the use of tables to gain the final mark.

17. Standardising and using the Normal tables to find the required probability in part (a)(i) caused few problems. The need to multiply this probability by 30 to find the expected number of jars was well understood but too many candidates assumed that this value had to be an integer. This is not the case and they lost one of the available marks. A clear diagram would have made the candidates realise that the appropriate z-value needed in part (b) was negative, –2.3263, this value being found in the table on page 22 of the booklet of tables and formulae. This type of calculation is still not understood by many of the candidates.

18. Far too many candidates were unable to make an attempt at this question. A good clear diagram would have helped candidates to see what was required of them. The use of the Normal distribution tables was very poor and where an attempt was made candidates often used a probability instead of a z-value showing that they really did not understand the use of the Normal distribution. Of those who could answer the question too many did not give their final answer to 3 significant figures.

19. Generally a well answered question but insistence on the use of –1.2816 and –1.96 caused some candidates to lose marks since their solutions were not always well presented. Candidates should be encouraged to draw a diagram when answering questions on the normal distribution and also to use the negative part of the z-axis. In part (b) a percentage was asked for but many candidates ignored this and consequently lost an easy mark.

20. No Report available for this question.